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Studies of the Optics of Neutrons. IV. Magnetic Saturation and Neutron Depolarization

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It is suggested that the magnetic saturation of ferromagnets should be studied by a slight modification of the previously used method. Instead of measuring the retardation of the single transmission effect in incompletely saturated ferromagnets as a function of the magnetizing field, it is recommended to study the depolarization of a totally polarized beam passing through a ferromagnet. This study would be carried out by a measurement of the polarization of the transmitted beam and no longer by an intensity measurement.

IN an earlier paper[1] by Holstein and the present author, the influence of magnetic fields in ferromagnets on transmitted neutrons was fully discussed. It was shown there that the small deviations from saturation within each Weiss domain accounted for the depolarization of the neutrons passing through, and thereby for a retardation of the single- and double-transmission effects. Formulas showing this retardation in the transmission effects were developed; it was also shown that the dependence of this retardation on the degree of saturation allowed one to draw conclusions about the law of magnetic saturation at high external field strength.

The first experimental investigation of the dependence of the single transmission effect, particularly near saturation, on the external magnetic field was carried out by Bloch[2] and collaborators. Their interest centered mostly on the investigation of how the polarization of neutrons was retarded by incomplete approach to saturation. Later on, shortly after the war, Hughes[2] and collaborators investigated what conclusions could be drawn from the behavior of the depolarization about the approach to saturation studied in its dependence on the external magnetic field. It was generally assumed previously that in agreement with the theory of crystalline forces, the approach to saturation follows in conventional notation the formula

$$I=I_0(1-a/H^2). \tag{1}$$

This result was not absolutely reliable since some measurements had given a saturation formula

$$I=I_0(1-b/H). \tag{2}$$

All this naturally referred to magnetometric investigations. Hughes *et al.*[2] found that within their limit of accuracy the saturation law was given by (2), for which we have, unlike (1), no theoretical basis; the data were evaluated according to the formulas of I.

Here an important point must be considered. The magnetometric and depolarization measurements do not measure exactly the same change of the magnetic moment in approach to saturation. The magnetic moment of an individual Weiss domain may change in absolute value, if only by a slight amount, if the external field becomes increasingly larger. To calculate this possible change would require a better knowledge of ferromagnetic theory than we possess at the moment. In addition, the magnetic moment becomes progressively better aligned parallel to the axis of the external magnetic field as saturation is approached.

The depolarization measurements, on the other hand, depend hardly at all on the possible slight increase of the value of the magnetic moment of the Weiss domain; as shown in I, depolarization is almost exclusively determined by the more or less imperfect alignment before saturation is reached. One must therefore expect that the two kinds of measurements would not give the same results; if there should be divergences it is easy to interpret them on the basis of the picture given in the foregoing remarks. The difficulty in measurement of the magnetic saturation law with the aid of neutron depolarization phenomena (for example, in the experiments by Hughes *et al.*) consists in the following: The single transmission effect ordinarily is small; one observes its changes (in dependence on the external magnetic field) which in turn become small as saturation is approached. In other words, the measurement of a small change of a small quantity is required and conclusions on the magnetic effect must be drawn from these measurements of small effects. This probably explains why the theoretically promising method was used only once by Hughes *et al.*[2] and why no later measurements have been published.

It is now proposed that use should be made of a certain advance in the technique of polarization which has been achieved since these experiments by Hughes *et al.*[2] were made. This advance consists of the following: We can now not only polarize with the aid of single- or double-transmission effects, a method which has a very poor output except for enormously thick samples, but it is also possible to polarize the incident beam almost completely with the aid of doubly-refracting mirrors or by selective reflection from a crystal. In a paper pub-

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[1] O. Halpern and T. Holstein, Phys. Rev. **59**, 960 (1941); this paper will hereafter be referred to as I.

[2] The reader is referred to a comprehenisve article on neutron optics by D. J. Hughes [Ann. Rev. Nucl. Sci. **3**, 93 (1953)]. This article not only treats the subject rather completely but also gives a detailed list of the various papers which had been published.

lished recently by Shull and Ferrier,[3] it was mentioned that polarizations of more than 99% could thus be obtained. The incident beam need no longer be unpolarized and become slowly partially polarized by transmission through the ferromagnet. It is, on the contrary, easily possible to start the experiments with a nearly totally polarized incident beam and observe its changes during transmission.

The procedure becomes very clear if we discuss the ideal case of a totally polarized beam falling on a ferromagnet which is made to attain various stages of saturation due to changes in the outside magnetic field. The beam, being already totally polarized, naturally will not now undergo any further polarization as in transmission effects, but will only lose a certain amount of polarization due to the depolarizing effects first discussed in I. When the beam emerges from the ferromagnet, its polarization is remeasured by one of many easily available methods and the depolarization, and thereby the law of magnetic saturation can be readily derived.

To put this scheme into very simple formulas: Let n_1 and n_2 denote, respectively, the number of incident neutrons in two polarization states which are so chosen that they propagate themselves through the ferromagnet with exponential decay. The exponential decay of n_1 is given by $e^{-(\omega+p)}$ per unit of length; that of n_2 correspondingly by $e^{-(\omega-p)}$; ω comprises scattering and absorption in the absence of a magnetic field; p the linear scattering due to the magnetic field. The existence of such laws has been proven in I and partially already before that in a paper by Halpern and Johnson.[2] Furthermore, there exists a probability coefficient α, also calculated in I, which indicates the scattering-free transition of a neutron from one state of polarization to the other. We thus have the following system of two linear differential equations for the population of the two polarization states:

$$dn_1/dx = -(\omega+p)n_1 - \alpha n_1 + \alpha n_2\,, \tag{3a}$$

$$dn_2/dx = -(\omega-p)n_2 - \alpha n_2 + \alpha n_1\,. \tag{3b}$$

By adding and subtracting, we obtain

$$d(n_1+n_2)/dx = -\omega(n_1+n_2) - p(n_1-n_2)\,, \tag{4a}$$

$$\begin{aligned} d(n_1-n_2)/dx = &-\omega(n_1-n_2) \\ &-p(n_1+n_2) - 2\alpha(n_1-n_2)\,. \end{aligned} \tag{4b}$$

Equations equivalent to (4a,b) have been derived in I in a much more cumbersome way; there they were also integrated for the purpose of calculating the effect of depolarization on transmission.

One can see, without reference to I and without any further calculation, but merely by inspection, that the effect on the polarization is much more pronounced for the initial condition $n_2=0$ than for the case so far investigated.

One measures, as stated before, the polarization when the beam leaves the ferromagnet; a change in polarization is due to the coefficient α, the dependence of which on the state of saturation is determined theoretically in I. We are here measuring sizeable effects rather than a small change in a small effect as before. Because of the simplicity of the method, we feel that Hughes *et al.* would have chosen it had well-polarized beams been available at the time of their experiments

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[3] C. J. Shull and R. P. Ferrier, Phys. Rev. Letters **10**, 295 (1963).